اطلاع قبل

$$\int \sec x\, dx = $$

$$\frac{1}{\cos x}\, dx = \int \frac{\cos x}{\cos^2 x}\, dx$$

$$= \int \frac{\cos x}{1-\sin^2 x}\, dx$$

$u = \sin x$

$$\int \frac{du}{1-u^2} = \frac{1}{2}\int \left(\frac{1}{1-u} + \frac{1}{1+u}\right) du$$

$$= \frac{1}{2}\left(-\ln(1-u) + \ln(1+u)\right) =$$

تذكير

$$\frac{1}{1-u} + \frac{1}{1+u} =$$

$$\frac{1+u+1-u}{1-u^2} = \frac{2}{1-u^2}$$

$$= \frac{1}{2}\left(\ln\frac{1}{1-\sin x} + \ln(1+\sin x)\right)$$

$$= \frac{1}{2}\ln\frac{1+\sin}{1-\sin} =$$

$$\ln\left(\frac{1+\sin}{1-\sin}\right)^{\frac{1}{2}} = \dots =$$

$$\int \sec x\, dx = \ln\left|\sec + \tan\right|$$ تذكير

$$\sec + \tan = \frac{1}{\cos} + \frac{\sin}{\cos} =$$

$$\frac{\cos + \cos\sin}{\cos^2} = \frac{\cos(1+\sin)}{(1-\sin)(1+\sin)}$$

$$= \frac{\cos}{1-\sin}$$

$$\sqrt{a^2-x^2}$$

$$\left(|x^2| < a^2 \quad -a \le x \le a \right)$$

$$-a \le x \le a$$

$$\sqrt{a^2-x^2} = \quad , \quad x = a\sin\theta \quad dx = a\cos\theta$$

$$= \sqrt{a^2 - a^2\sin^2\theta} = \sqrt{a^2(1-\sin^2)} = \quad a\sqrt{\cos^2} = a\cos\theta$$

$$\sqrt{x^2-a^2}$$

تغییر متغیرهای مثلثاتی

برای [illegible] انتگرال‌های شامل

$$\sqrt{x^2+a^2} \quad \text{و} \quad \sqrt{a^2-x^2} \quad ,$$

$$\int \tan^2 \sec x\, dx =$$

$$\int (\sec^2 - 1)\sec x\, dx =$$

$$\int \left(\sec^3 - \sec dx\right)$$

$$= \ldots$$

$$\sec x \tan x - \int (\tan)(\sec \tan)$$

$$\sec' = \left(\frac{1}{\cos}\right)' = \frac{+\sin}{\cos^2} =$$

$$\frac{\sin}{\cos} \times \frac{1}{\cos} = \sec \tan$$

$$\int \sec^3 dx = \int \frac{1}{\cos^3 x} dx$$

$$= \int_{u=\sec} \frac{1}{\cos x} \times \frac{1}{\cos^2 x} dx = \quad (dv)$$

ادامه ح قبل

① $\int \sec\, dx = \ln|\sec + \tan|$

② $\int \sec^2 dx = \tan$

③ $\int \sec^3 x\, dx$ [illegible]

مثال

$$\int \sqrt{1-x^2}\,dx$$

$$x = \sin\theta \Leftrightarrow \theta = \sin^{-1}(x)$$

$$dx = \cos\theta\, d\theta$$

$$\int \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta =$$

$$\int \cos\theta\cos\theta\, d\theta = \int \cos^2\theta\, d\theta$$

$$= \int \left(\frac{1+\cos 2\theta}{2}\right) d\theta =$$

$$\int \frac{1}{2}\, d\theta + \int \frac{1}{2}\cos 2\theta\, d\theta$$

$$= \frac{1}{2}\theta + \frac{\sin 2\theta}{4} =$$

$$\frac{1}{2}\sin^{-1}(x) + \frac{\sin(2\sin^{-1}x)}{4}$$

توجه

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \Rightarrow$$

$$-1 \leq \sin\theta \leq 1$$

مثال

$$\int \frac{1}{\sqrt{4-x^2}}\, dx$$

راه حل قدیم:

$$= \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\, dx = \sin^{-1}\left(\frac{x}{2}\right)$$

راه حل جدید:

$$x = 2\sin\theta$$

$$dx = 2\cos\theta\, d\theta$$

$$\int \frac{2\cos\theta\, d\theta}{\sqrt{4-4\sin^2\theta}} =$$

$$\int \frac{2\cos\theta\, d\theta}{2\cos\theta} = \int d\theta$$

$$\frac{x}{2} = \sin\theta$$

$$= \theta = \sin^{-1}\left(\frac{x}{2}\right)$$

$(-\cot x)' = \cot^2 x + 1$

$\int \cot^2\theta \, d\theta = \int (\cot^2 + 1 - 1) \, d\theta$

$= -\cot\theta - \theta =$

$-\cot\left(\sin^{-1}\frac{x}{3}\right) - \sin^{-1}\frac{x}{3}$

$= \int \cot^2\theta \, d\theta =$

$(\cot x)' = \left(\frac{\cos}{\sin}\right)' =$

$\frac{-\sin^2 - \cos^2}{\sin^2} = \frac{-\sin^2}{\sin^2} - \frac{\cos^2}{\sin^2} =$

$-1 - \cot^2\theta$

$\int \frac{\sqrt{9 - x^2}}{x^2} \, dx =$

$x = 3\sin\theta$

$\int \frac{3\cos\theta}{9\sin^2\theta} \times 3\cos\theta \, d\theta$

$\int x\sqrt{9 - x^2} \, dx$

$u = 9 - x^2$

$du = -2x \, dx$

$-\frac{1}{2}\int u^{\frac{1}{2}} \, du = \dots$

$\int \frac{\sqrt{9 - x^2}}{x^2} \, dx$

$\int x\sqrt{9 - x^2} \, dx$

$$\int \frac{1}{\sqrt{1+x^2}}\, dx = ?$$

حل

$x = \tan\theta$

$dx = (1+\tan^2\theta)\, d\theta$

$= \sec^2\theta\, d\theta$

$$= \int \frac{\sec^2\theta}{\sqrt{1+\tan^2\theta}}\, d\theta = \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta| = \ln|\sec(\tan^{-1}x) + x|$$

$$\int \sqrt{x^2-1}\, dx$$

تمرین

$$\sqrt{x^2+a^2}$$

تمرین

$Dom = \mathbb{R}$

$x = a\tan\theta$

$$\int \frac{\sec\theta\tan\theta}{\sec\theta\tan\theta}\, d\theta$$

$$= \int d\theta = \theta = \sec^{-1}(x)$$

$x = \frac{1}{\cos\theta} = \sec\theta$

$\theta = \sec^{-1}x$

$$\int \frac{1}{x\sqrt{x^2-1}}\, dx = ?$$

حل

$x = \frac{1}{\cos\theta} = \sec\theta$

$dx = \sec\theta\tan\theta$

$$\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \tan$$

$$\sqrt{x^2-1}$$

$x^2 \geq 1$

$x \in (-\infty,-1] \cup [1,+\infty)$

$\frac{1}{\cos}$

$-1 \leq \cos\theta \leq 1 \Rightarrow \frac{1}{\cos} \geq 1$

$\frac{1}{\cos} \leq -1$

$$\sqrt{x^2-a^2}$$

تمرین

$(-\infty,-a] \cup [a,+\infty)$

$x \leq -a$

$x \geq a$

$x = \frac{a}{\cos} = a\sec\theta$

تمرین

$$\int \frac{x^3}{(4x^2+9)^{\frac{3}{2}}}\,dx$$

مثال

$$\int \frac{x}{\sqrt{x^2+4}}\,dx \longrightarrow$$

$u = x^2+4$

$$\int \frac{du}{2u^{\frac{1}{2}}} = \dots$$

$$\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta \qquad u=\sin\theta$$

$$= \frac{1}{4}\int \frac{du}{u^2} = \frac{1}{4}\int u^{-2}\,du$$

$$= -\frac{1}{4}\,u^{-1} = -\frac{1}{4}(\sin\theta)^{-1} =$$

$$-\frac{1}{4}\left(\sin\left(\tan^{-1}\frac{x}{2}\right)\right)^{-1}$$

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta =$$

$$= \frac{1}{4}\int \frac{1}{\cos\theta \times \frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$= \frac{1}{4}\int \frac{1}{\frac{\sin^2\theta}{\cos\theta}}\,d\theta =$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$$\int \frac{2\sec^2\theta\,d\theta}{4\tan^2\theta \times 2\sec\theta}$$

$$\sqrt{a^2+a^2\tan^2} = a\sqrt{\sec^2}$$

تمرین

مثال قبل را با تغییر متغیر $x=\sinh y$ حل کنید

مثال

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$

$$= \int (2\sin\theta - 1)\, d\theta$$

$$= \cdots$$

$$\int \frac{x\,dx}{\sqrt{4-(x+1)^2}}$$

$x+1 = 2\sin\theta$ $\quad dx = 2\cos\theta\, d\theta$

$$\int \frac{(2\sin\theta - 1)\, 2\cos\theta}{2\cos\theta}\, d\theta$$

$$\sqrt{3-2x-x^2} = \sqrt{4-1-2x-x^2} = \sqrt{4-(x+1)^2}$$

$$\int \frac{x\,dx}{\sqrt{3-2x-x^2}} = \int \frac{x\,dx}{\sqrt{4-(x+1)^2}}$$