روشهای انتگرال گیری

برآدی

$n \neq 1$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

$$\int e^x\,dx = e^x$$

$$\int b^x\,dx = \frac{b^x}{\ln b}$$

$$\int \sin x\,dx = -\cos x$$

$$\int \cos x\,dx = \sin x$$

$$\int \sec^2 x\,dx = \tan x$$

$$\int \sec x \tan x = \sec x$$

$$(\sec)' = \left(\frac{1}{\cos}\right)' = \frac{\sin}{\cos^2} = \frac{\sin}{\cos} \times \frac{1}{\cos}$$

$$\int \sec x\,dx = ?$$

بعداً

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \ln|\cos x|$$

$$\int \sinh x\,dx = \cosh x$$

$$\int \cosh x\,dx = \sinh x$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right)$$

روشهای انتگرال گیری

یادآوری

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx =$$

($u = \cos x$, $-du$)

$$-\ln|\cos x| = \ln\frac{1}{|\cos x|}$$

مشتق

$$\left(u(x)v(x)\right)' = u'(x)v(x) + v'(x)u(x)$$

$$\frac{d(uv)}{dx} = \frac{du}{dx}\times v + \frac{dv}{dx}u$$

$$d(uv) = u\,dv + v\,du$$

$$u\,dv = d(uv) - v\,du$$

پس

$$\int u\,dv =$$

$$\int d(uv) - \int v\,du =$$

$$uv - \int v\,du$$

خلاصه

$$\int u\,dv = uv - \int v\,du$$

$$\int x \sin x\,dx$$

($u = x$, $dv = \sin x\,dx$)

$du = dx$

$v = (-\cos x)$

مثال

$dv = \sin x\,dx$

$\frac{dv}{dx} = \sin x$

$v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$= x(-\cos x) - \int(-\cos x)\,dx$$

$$= -x\cos x + \sin x + C \;\checkmark$$

راه حل دوم

$$\int x \sin x\,dx = uv - \int v\,du$$

($u = \sin x$, $dv = x\,dx$)

$$= \sin x \times \frac{x^2}{2} - \int +\frac{x^2}{2}\cos x\,dx$$

که سخت!

روش فوق را روش جزء به جزء می نامیم

Integration by parts

روشهای انتگرال‌گیری

مثال

$$\int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv} = ?$$

$$v = x$$

$$\int u\,dv = uv - \int v\,du =$$

$$\int \ln x\,dx = x\ln x - \int x \times \frac{1}{x}\,dx$$

$$= x\ln x - x = x(\ln x - 1)$$

$$d(uv) = u\,dv + v\,du$$

$$\int u\,dv = uv - \int v\,du$$

مثال

$$\int \underbrace{x}_{u}\, \underbrace{e^x dx}_{dv} = ? \qquad v = e^x$$

$$= xe^x - \int e^x dx =$$

$$xe^x - e^x = (x-1)e^x$$

مثال

$$\int \underbrace{x^2}_{u} e^x\,dx = ?$$

$$= x^2 e^x - \int 2e^x x\,dx$$

$$= x^2 e^x - 2(x-1)e^x$$

مثال

$$\int \underbrace{\arctan x}_{u}\, \underbrace{dx}_{dv} = ?$$

$$= x\tan^{-1}(x) - \int x \times \frac{1}{1+x^2}\,dx$$

$$= x\tan^{-1}x - \frac{\ln(1+x^2)}{2} + c$$

روش اول

$$\int \sin^3 x \, dx =$$

$$\int (\sin^2 x) \sin x \, dx = \int (1-\cos^2 x) \sin x \, dx$$

$u = \cos x$

$du = -\sin x$

$$= \int (u^2-1) \, du = \frac{u^3}{3} - u = \frac{\cos^3 x}{3} - \cos x$$

$$= -\cos x \sin^2 x + 2\left(-\cos^3 x - \int + \cos x \, 2 \sin x \cos x\right)$$

$$= -\cos x \sin^2 x - 2\cos^3 x - 4\int \cos^3 x \sin x \, dx$$

$$= -\cos x \sin^2 x - 2\cos^3 x - 4\frac{\cos^3 x}{3}$$

$$\int \sin^3 x \, dx = \int \underbrace{\sin^2 x}_{u} \underbrace{\sin x \, dx}_{dv}$$

$$= \sin^2 x(-\cos x) - \int (-\cos x) \, 2 \sin x \cos x \, dx$$

$$= -\cos x \sin^2 x + 2\int \underbrace{\cos^2 x}_{u} \underbrace{\sin x \, dx}_{dv}$$

توجه

$$\int \sin x \, dx = -\cos x$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$$

$$\frac{1+\cos 2x}{2} = \cos^2 x$$

$$\frac{1-\cos 2x}{2} = \sin^2 x$$

تمرین

$$\int \frac{x}{1+x^2} \, dx = \frac{1}{2}\int \frac{du}{u}$$

$u = 1+x^2$

$du = 2x \, dx$

$$= \frac{\ln|u|}{2} = \frac{1}{2}\ln(x^2+1)$$

مثال

$$\int e^x \sin x\,dx = -\cos x\, e^x + \sin x\, e^x - \int e^x \sin x\,dx$$

$$\Rightarrow 2\int e^x \sin x\,dx = e^x(\sin x - \cos x)$$

$$\Rightarrow \int e^x \sin x\,dx = \frac{e^x}{2}(\sin x - \cos x)$$

مثال (حل دوم)

$$\int \underbrace{e^x}_{u} \underbrace{\sin x\,dx}_{dv}$$

$$= e^x(-\cos x) + \int (\cos x)\underbrace{e^x}_{u}\,dx$$

$$= e^x(-\cos x) + \left[e^x \sin x - \int \sin x\, e^x\,dx \right.$$

$$\int \sin^2 x \cos^2 x\,dx = \int (\sin x \cos x)^2\,dx$$

$$= \int \left(\frac{\sin 2x}{2}\right)^2 dx = \cdots$$

مثال

$$\int \sin^4 x\,dx = ?$$

$$\int \underbrace{\sin^3 x}_{u} \underbrace{\sin x\,dx}_{dv} = (-\sin^3 x \cos x) - \int \cos x \cos x\, 3\sin^2 x$$

$$= -\sin^3 x \cos x - \boxed{3\sin^2 x \cos^2 x\,dx}$$

تمرین

$$\int \sin^n x\,dx = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

توان زوج

$$\int \sin^{2m} x \, dx \rightarrow$$

$$\int \sin^4 x \, dx =$$

$$\int \left(\sin^2\right)^2 dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int \left(1 + \cos^2 2x - 2\cos 2x\right) dx$$

$$= \dots$$

توان زوج

$$\int \sin^2 x \, dx =$$

$$\int \frac{1-\cos 2x}{2} \, dx = \frac{1}{2}x - \frac{\sin 2x}{4}$$

$$\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} \, dx = \dots$$

$$\int \left(\sin^{2m+1}\right) dx$$

$$\int \sin^5 x \, dx = \int \sin^4 x \sin x \, dx =$$

$$= \int \left(\sin^2\right)^2 \sin x \, dx = \int \left(1-\cos^2\right)^2 \sin x \, dx$$

$$= -\int (1-u^2)^2 du = \dots$$

$$u - \frac{u^3}{3} = \sin x - \frac{1}{3}\sin^3 x$$

$$\int \sin^3 x \, dx = \int \sin^2 x \sin x \, dx$$

$$= \int \left(1-\cos^2 x\right) \sin x \, dx =$$

$$\int (u^2-1) \, du = \dots$$

اگر [illegible] متناظر

مثال

$$\int \cos^3 x \, dx =$$

$$\int \left(\cos^2 x\right) \cos x \, dx = \int \left(1-\sin^2 x\right) \cos x \, dx$$

$$u = \sin x$$

$$= \int (1-u^2) \, du$$

مثال

$$\int \sin^{5}x \cos^{2}x \, dx =$$

$$\int \sin^{4}x \cos^{2}x \sin x \, dx =$$

$$\int (\sin^2)^2 \cos^2 \sin x \, dx =$$

$$= \int (1-\cos^2)^2 \cos^2 \sin x \, dx$$

$u = \cos x$

$$\int u^2 (1-u^2)^2 \, du = \ldots$$

مثال

$$\int \tan^{6}x \sec^{4}x \, dx =$$

$$= \int \tan^{6}x \times \sec^{2} \times \sec^{2} \, dx =$$

$$\int (\tan^2)^3 \times (\sec)^2 \times \sec^2 \, dx \ldots$$

$$= \int (\tan^2)^3 (1+\tan^2) \sec^2 \, dx$$

$u = \tan$

$$= \int (u^2)^3 (1+u^2) \, du$$

$$\int \tan^{5} \sec^{7} \, dx =$$

$$\int \tan^{5} \sec^{6} \sec \, dx =$$

$$\int \tan^{4}x (\sec^{6}x) \times \tan x \sec x \, dx$$

$$= \int (\tan^2)^2 \sec^6 \sec \tan \, dx$$

$$= \int (\sec^2 - 1)^2 \sec^6 (\sec \tan) \, dx$$

$$= \int (u^2-1)^2 u^6 \, du = \ldots$$

$$2\int \sec^3 x\,dx = \sec\tan x + \int \sec x\,dx$$

$$\int \sec x\,dx = ?$$

حل

$$= (\sec x)\tan x - \int (\tan x)\sec\tan dx$$

$$= \sec x\tan x - \int \tan^2 \sec x\,dx$$

$$= \sec x\tan x - \int (\sec^2 - 1)(\sec)\,dx$$

$$= \sec x\tan x - \int (\sec)^3 dx + \int \sec x\,dx$$

$$\int \sec^2 x\,dx = \tan x$$

$$\int \sec^3 x\,dx = \int \underbrace{(\sec x)}_{u}\underbrace{(\sec^2 x)\,dx}_{dv}$$

$$= \int \underbrace{\tan}_{u}\underbrace{\sec^2}_{du} dx - \int \tan x\,dx$$

$$\frac{\tan^2 x}{2} + \ln(|\cos x|) + C$$

حل (م)

$$\int \tan^3 x\,dx = \int (\tan^2 x)\tan x\,dx$$

$$= \int (\sec^2 - 1)\tan dx =$$