یادآوری

تابع نمایی

$b^r := e^{r\ln b}$

$b>0$

$\sqrt{3}^{\pi} = e^{\pi \ln\sqrt{3}}$

$b^{r_1+r_2} = b^{r_1} \times b^{r_2}$ $\quad b>0$

$b^{r_1+r_2} = e^{(r_1+r_2)\ln b} = e^{r_1\ln b} \times e^{r_2 \ln b} = b^{r_1} \times b^{r_2}$

$(b^{r_1})^{r_2} = b^{r_1 r_2}$

توجه

$(b^x)' = (e^{x\ln b})' = \ln b \times e^{x\ln b} = \ln b \times b^x$

$(b^x)' = b^x \ln b$

$\int b^x dx = \frac{b^x}{\ln b} + C$

$\int_{\cdot}^{5} 2^x dx$

$\frac{2^x}{\ln 2}\Big|_{\cdot}^{5} = \frac{2^5}{\ln 2} - \frac{1}{\ln 2}$

$2^x \neq x^2$

$$y' = \left(e^{x\ln x}\right)' =$$

$$\left(\ln x + \frac{1}{x} \times x\right) \times e^{x\ln x} =$$

$$(\ln x + 1) \times e^{x\ln x}$$

تعیین علامت

$y = x^{x}$ لازم کنید

$e^{x} > 0$

$y = e^{x\ln x}$

دامنه: $(0, +\infty)$

تمرین

$$u = e^{x}$$

$$du = e^{x}\,dx$$

$$\int \frac{e^{x}\,dx}{e^{2x}+1}$$

حل

$$\int \frac{e^{x}\,dx}{(e^{x})^{2}+1} = \int \frac{du}{u^{2}+1}$$

$$= \int \frac{du}{u \ln 2} = \frac{1}{\ln 2}\int \frac{du}{u} = \frac{\ln u}{\ln 2}$$

$$= \frac{\ln(2^{x}+1)}{\ln 2}$$

حل

$$\int \frac{2^{x}}{2^{x}+1}\,dx$$

$$u = 2^{x} + 1$$

$$du = 2^{x}\ln 2\,dx$$

$$= \lim_{u \to +\infty} e^{-u} \times (-u)$$

$$= \lim_{u \to +\infty} \frac{-u}{e^{u}}$$

$$= \lim \frac{-1}{e^{u}} = 0$$

$0 \times -\infty$

$$\lim_{x \to 0^+} x \ln x = \lim_{t \to -\infty} e^{t} \times t$$

$x = e^{t}$

$t \to -\infty$

$t = -u$

$u \to +\infty$

$$\lim_{x \to 0^+} x^{x} =$$

$$\lim_{x \to 0^+} e^{x \ln x}$$

$$\lim_{x \to 0^+} x \ln x$$

$y' = 0 \Rightarrow \ln x + 1 = 0$

$\Rightarrow \ln x = -1 \Leftrightarrow$

$x = e^{-1} = \frac{1}{e}$

$2.71..$

$x = \frac{1}{e} \Rightarrow \ln x + 1 = 0$

$x < \frac{1}{e} \Rightarrow \ln x + 1 < 0$

$x > \frac{1}{e} \quad \ln x + 1 > 0$

$$\lim_{x\to+\infty} x\ln x = \lim_{t\to+\infty} e^t\, t = +\infty$$

$x = e^t$

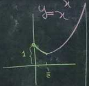
$t\to+\infty$

ادامه

$$\lim_{x\to+\infty} x^x = \lim_{x\to+\infty} e^{x\ln x}$$

ابتدا

$$\lim_{x\to+\infty} x\ln x$$



ادامه جواب سوال

$$\lim_{x\to0^+} x^x = \lim_{x\to0^+} e^{x\ln x} =$$

$$e^{\lim_{x\to0^+} x\ln x} = e^0 = 1$$

$$\lim_{x\to+\infty} \frac{x^3}{e^x} = 0$$

⋮

توجه

$$\lim_{x\to+\infty} \frac{x}{e^x} = 0$$

$$\lim_{x\to+\infty} \frac{x^2}{e^x} = 0$$

...

رسم کنید

$x^{\sqrt{x}}$

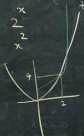
$x^{\sin x}$

$x^{\frac{1}{x}}$

$x^{\frac{1}{x^2}}$

2^x

x^2



$$\lim_{x \to +\infty} \frac{x^n}{e^x} = 0$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

x^x

$$f''(x) = \frac{1}{x} e^{x\ln x} + (\ln x+1)^2 e^{x\ln x}$$

$$= e^{x\ln x}\left(\frac{1}{x} + (\ln x+1)^2\right)$$

$>0 \quad >0 \quad >0$

$\forall x>0 \quad f''(x) > 0$

$$f(x) = x^x = e^{x\ln x}$$

$$f'(x) = (\ln x + 1)\, e^{x\ln x}$$

$$\lim_{x\to 0^+} \frac{\ln(x+1)}{x} = 1$$

$$\lim_{x\to 1} \frac{\ln x}{x-1} = 1$$

$$\lim_{x\to 0^+} e^{\frac{1}{x}\ln(1+x)} = e$$

$$e = \lim_{x\to 0^+} (1+x)^{\frac{1}{x}}$$

پاسخ

$$\lim_{x\to 0^+} (1+x)^{\frac{1}{x}} = \lim_{x\to 0^+} e^{\frac{1}{x}\ln(1+x)}$$

$$\lim_{x\to 0^+} \frac{\ln(1+x)}{x} = \lim_{h\to 0^+} \frac{\ln(1+h)-\ln 1}{h}$$

$$= (\ln)'(1) = 1$$

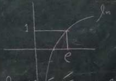
مثال

$$\lim_{x\to 0^+} (1+x)^{\frac{1}{x}}$$

توجه

1^{∞} مبهم است.

یادآوری



e عددی است که در آن تابع ln برابر با یک می‌شود

توجه

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n} = e$$

$\frac{1}{n} = t$

$$\lim_{t\to 0^+}(1+t)^{\frac{1}{t}}$$

$$\left(1+\frac{1}{2}\right)^{2} = 1+\frac{1}{4}+1$$

$$\left(1+\frac{1}{3}\right)^{3} = \cdots$$

$$\left(1+\frac{1}{4}\right)^{4}$$

$$\left(1+\frac{1}{5}\right)^{5}$$

$\to e$ $\quad \sum \frac{x^n}{n!}$

رفع ابهام از 1^∞

فرض کنید

$$\lim_{x\to a} f(x) = 1$$

$$\lim_{x\to a} g(x) = +\infty$$

آنگاه

$$\lim_{x\to a} f(x)^{g(x)} =$$

$$\lim_{x\to a}$$ e [redacted]

$g(x)(f(x)-1)$

$$\lim_{x\to 0^+}(1+x)^{\frac{1}{x}} =$$

$$\lim_{x\to 0^+} e^{\frac{1}{x}(1+x-1)}$$

$$= \lim_{x\to 0} e^{1} = e$$

اثبات

$$\lim_{x\to a} f(x)^{g(x)} = \lim_{x\to a} e^{g(x)\ln f(x)}$$

$$= \lim_{x\to a} e^{g(x)\left(\frac{\ln f(x)}{f(x)-1}\right)\times(f(x)-1)}$$

$$= \lim_{x\to a} e^{g(x)(f(x)-1)}$$

$$\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x}$$

$$\left(1+\frac{1}{x+1}\right)^{x}$$

حل

$$\lim_{x\to 0^+}\left(1+ax\right)^{b/x}$$

$$=\lim_{x\to 0^+} e^{\frac{b}{x}(ax)} = e^{ab}$$