$$\lim_{x\to-\infty} e^{x} = \lim_{t\to 0^+} e^{\ln t} = \lim_{t\to 0^+} t = 0$$

$x=\ln t$ $\quad (x\to-\infty \Leftrightarrow t\to 0^+)$

$$\lim_{x\to+\infty} e^{x} = \lim_{t\to+\infty} e^{\ln t}$$

$x=\ln t$

$$= \lim_{t\to+\infty} t = +\infty$$

$e^{0}=1$ پس

$\ln 1 = 0$ زیرا

$e^{\ln t}=t$

$e^{a}=b \Leftrightarrow \ln b = a$

$\ln(e^{t})=t$ لگاریتم طبیعی

$(a,b)\in \exp$

~~$\ln x \times \ln y \neq \ln xy$~~

$e^{x} = f(x)$ را با $\ln x$ معکوس تابع

نمایش می‌دهیم

یادآوری

$y=\ln x$

$(\ln x)' = \frac{1}{x}$

$\ln 1 = 0$

$(\ln xy = \ln x + \ln y)$

نتیجه

$$\ln e^{a} = a = a\ln e$$

اثبات

باید ثابت کنیم که $\ln(e^{a} \times e^{b}) = a+b$

$$\ln(e^{a}) + \ln(e^{b}) =$$

$$a\ln e^{1} + b\ln e^{1} = a+b$$

یادآوری

$$e^{x} : (-\infty,+\infty) \to (0,+\infty)$$

$$\boxed{e^{a+b} = e^{a} \times e^{b}}$$

$$e^{c} = d \Leftrightarrow \ln d = c$$

خلاصه

$$(e^{x})' = e^{x}$$

$$(e^{y})'_{(y)} = \frac{1}{(\ln x)'} = x = e^{y}$$

$$(\ln x)' = \frac{1}{x}$$

$$(y = \ln x) \Updownarrow e^{y} = x$$

یادآوری

$$(f^{-1}(y))' = \frac{1}{f'(x)}$$

$$(y = f(x))$$

$$= \lim_{x \to +\infty} \frac{\frac{e^{2x}}{e^{2x}}}{\frac{e^{2x}}{e^{2x}} + \frac{1}{e^{2x}}} =$$

$$= \lim_{x \to +\infty} \frac{1}{1 + \frac{1}{e^{2x}}} = \frac{1}{1} = 1$$

$$\lim_{x \to +\infty} \frac{e^{2x}}{e^{2x} + 1}$$

$e^{2x} \to +\infty$, $e^{2x}+1 \to +\infty$

$f(x) = x$, $g(x) = x$: $\lim_{x \to +\infty} \frac{f(x)}{g(x)} = \lim_{x \to +\infty} \frac{x}{x} = 1$

$f(x) = x$, $g(x) = 2x$: $\lim_{x \to +\infty} \frac{f(x)}{g(x)} = \lim_{x \to +\infty} \frac{x}{2x} = \frac{1}{2}$

مثل

$$e^{a} = b \Leftrightarrow \ln b = a$$

$$\left(e^{r}\right)^{x} = e^{rx}$$

لأن

$$\ln\left(e^{r}\right)^{x} = x \ln\left(e^{r}\right)$$

$$= rx \ln e = rx$$

$$e^{0} = 1$$

$$e^{a+b} = e^{a} \times e^{b}$$

$$e^{(a-b)} = \frac{e^{a}}{e^{b}}$$

$$\ln\left(\frac{e^{a}}{e^{b}}\right) = \ln e^{a} - \ln e^{b} = a - b$$

$$(ue^u)' = e^u + ue^u$$

$$\int ue^u\,du = ue^u - e^u$$

$$\int \ln x\,dx = (\ln x \times x) - x$$

$$\int \ln x\,dx = ?$$

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$dx = x\,du = e^u\,du$$

مثال

$$\int e^x\,dx = e^x + c$$

$$\int \frac{\ln x}{x}\,dx$$

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$\int u\,du = \frac{u^2}{2} + c = \frac{(\ln x)^2}{2} + c$$

مثال

$$\left(e^{\tan x}\right)' = (\sec^2 x)\,e^{\tan x}$$

$$\left(e^{-4x}\sin x\right)' = -4e^{-4x}\sin x + \cos x\,e^{-4x}$$

$$\left(e^{f(x)}\right)' = f'(x)\,e^{f(x)}$$

مثال

مشتق توابع زیر را حساب کنید:

$$e^{-4x}\sin x$$

$$(e^x)' = e^x$$

$$=\int x^2 e^{x^3} dx =$$

$$=\int e^u \frac{du}{3} =$$

$$\frac{1}{3}\int e^u du = \frac{1}{3} e^u + c$$

$$= \frac{1}{3} e^{x^3} + c$$

$$\int x^2 e^{x^3} dx =$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$x^2 dx = \frac{du}{3}$$

حل

$$\int \ln x \, dx = \int u e^u du \longrightarrow$$

$$u = \ln x$$

$$du = \frac{1}{x} dx \Rightarrow dx = x\,du = e^u du$$

$$\int e^x dx = e^x + c$$

$$\int x e^x dx = e^x(x-1) + c$$

$$\int \ln x \, dx = x \ln x - x$$

مرور

$$(xe^x)' = e^x + e^x x$$

$$(xe^x - e^x)' = e^x + xe^x - e^x = xe^x$$

مثال

تابع $e^{\frac{1}{x}}$ را رسم کنید.

$$f(x) = e^{\frac{1}{x}}$$

$f(0)$ تعریف نشده

$$\text{Dom} f = \mathbb{R} - \{0\}$$

$$f'(x) = -\frac{1}{x^2} e^{\frac{1}{x}} < 0$$

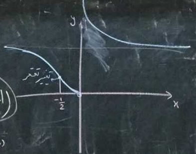
پس تابع f در دامنه خود نزولی است.

$$\lim_{x \to +\infty} f(x) = 1$$

$$\lim_{x \to 0^+} f(x) = +\infty$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} e^{\frac{1}{x}} = 0$$

$$\lim_{x \to -\infty} f(x) = 1$$



$$f'(x) = -\frac{1}{x^2} e^{\frac{1}{x}}$$

$$f''(x) = \frac{2x}{x^4} e^{\frac{1}{x}} + \frac{1}{x^2} e^{\frac{1}{x}} \left(\frac{1}{x^2}\right)$$

می‌شود تقعر

$$\frac{2x}{x^4} e^{\frac{1}{x}} + \frac{1}{x^4} e^{\frac{1}{x}} = \frac{e^{\frac{1}{x}}}{x^4}(2x+1)$$

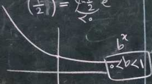
در $x = -\frac{1}{2}$ مشتق دوم صفر است

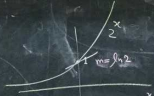
$x > -\frac{1}{2}$ مشتق دوم مثبت

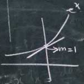
$x < -\frac{1}{2}$ مشتق دوم منفی

$\left(\frac{1}{2}\right)^x = e^{x\ln\frac{1}{2}}$ $\quad \ln\frac{1}{2} < 0$

$\left(\left(\frac{1}{2}\right)^x\right)' = \underbrace{\ln\frac{1}{2}}_{<0}\, e^{x\ln\frac{1}{2}}$







$\ln 2 > 0$

$$(2^x)' = \boxed{\ln 2 \times e^{x\ln 2}}$$

$$= \ln 2 \times 2^x$$

$$2^x = e^{x\ln 2}$$

$$2^3 = e^{3\ln 2} = e^{\ln 8} = 8$$

$$2^\pi = e^{\pi \ln 2}$$

تعاريف

$$b^x := e^{x\ln b}$$

$(b>0)$ $\quad x\in(-\infty,+\infty)$