زوج

$$\int \sec x \, dx = \int \frac{1}{\cos x} \, dx$$

$$\int \sec x \, dx = \ln \left| \sec x + \tan x \right|$$

$$\int \sec x \, dx = \int \frac{1}{\cos x} \, dx =$$

$$\int \frac{\cos x}{\cos^2 x} \, dx = \int \frac{\cos x \, dx}{1 - \sin^2 x}$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{du}{1-u^2} = \frac{1}{2} \int \left(\frac{1}{1-u} + \frac{1}{1+u} \right) du$$

$$= \frac{1}{2} \left(-\ln|1-u| + \ln|1+u| \right) = \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C$$

$$= \frac{1}{2} \ln \left| \frac{1+\sin x}{1-\sin x} \right| + C$$

$$= \ln \left| \sec x + \tan x \right| + C$$

یادآوری

$$d(uv) = u\,dv + v\,du$$

$$uv = \int u\,dv + \int v\,du$$

حل

$$\int u\,dv = uv - \int v\,du$$

$$\int x e^{x^2} dx = \frac{1}{2}\int e^{u} du = \frac{1}{2} e^{x^2} + c$$

حل

$u = x^2$

$du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$$\int \underset{u}{x}\, \underset{dv}{e^{x} dx}$$

$$uv - \int v\,du$$

$v = e^{x}$

$u = x$

$$\int x e^{x} dx = x e^{x} - \int e^{x} dx = x e^{x} - e^{x} + C = (x-1)e^{x} + c$$

~~$$\int x \underset{dv}{e^{x}} dx = \frac{x^2}{2} e^{x} - \int \frac{x^2}{2} e^{x} dx$$~~

$x^2 e^x - 2\int x e^x \, dx$

$= x^2 e^x - 2\left((x-1)e^x\right) + c$

$\int x^2 e^x \, dx = x^2 e^x - \int e^x \, 2x \, dx$

$\int x e^x \, du$

$u = x^2$

$dv = e^x dx$

$v = e^x$

$2(u-1)e^u + c =$

$2(\sqrt{x} - 1) e^{\sqrt{x}} + c$

$\int e^{\sqrt{x}} \, dx = 2\int u e^u \, du$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx \Rightarrow dx = 2u\,du$

$$\Rightarrow \int e^x \sin x\, dx = \frac{e^x(\sin x - \cos x)}{2} + C$$

$$2\int e^x \sin x\, dx = e^x(\sin x - \cos x)$$

$$\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx$$

$$= e^x \sin x - \left[e^x \cos x + \int e^x \sin x\, dx \right]$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x\, dx$$

$$\int e^x \cos x\, dx = uv - \int v\, du$$

u, du

$$= \cos x\, e^x + \int e^x \sin x\, dx$$

$$\int e^x \sin x\, dx = \frac{du}{}$$

u, du

$$uv - \int v\, du = \sin x\, e^x - \int e^x \cos x\, dx$$

$v = e^x$

$u = \sin x$

$= x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + c$

$x\tan^{-1}(x) - \int \frac{x\,dx}{1+x^2}$ t

$dt = 2x\,dx$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{dt}{t} =$

$= x\tan^{-1}x - \frac{1}{2}\ln|t| + c$

$\int \arctan x \, dx =$ u $d\varphi$

$u = \tan^{-1}x$

$\int \tan^{-1}x\,dx$ $v = x$

$\int \sin(u)\, e^u\, du =$

$\frac{e^u(\sin u - \cos u)}{2} + c =$

$\frac{e^{\ln x}(\sin\ln x - \cos\ln x)}{2} + c = x\frac{(\sin\ln x - \cos\ln x)}{2} + C$

$\int \sin(\ln x)\,dx$

$u = \ln x$

$du = \frac{1}{x}dx \Rightarrow dx = x\,du = e^u du$

$\int u \cos u \, du =$

راه حل دوم تمرین ۲

استفاده از رابطه تکراری

راه حل دوم تمرین ۱

$\int \arcsin x \, dx$

$u = \arcsin x$

$\sin u = x \quad dx = \cos u \, du$

$-\frac{1}{2}\int t^{-\frac{1}{2}} dt =$

$-\frac{1}{2} \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = -\frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c$

حل

$\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} =$

$t = 1-x^2$

$dt = -2x\,dx \Rightarrow x\,dx = \frac{dt}{-2}$

$\int \underbrace{\arcsin x}_{u} \underbrace{dx}_{dv}$

$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$

حل

$$\int (\ln x)^2 \, dx$$

تمرین

$$\int x \ln x \, dx$$ ($u = \ln x$, dv) حل

$$= \ln x \frac{x^2}{2} - \int \frac{x^2}{2} \times \frac{1}{x} dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{4} x^2 + C$$

$$\int \ln x \, dx = x \ln x - \int x \times \frac{1}{x} dx = x \ln x - x + C$$ حل

$$= x \sin x - \int \sin x \, dx =$$

$$x \sin x + \cos x + C$$

$$\int x \sin x \, dx = ?$$ تمرین

$$\int x \cos x \, dx = uv - \int v \, du$$ ($u = x$, $dv = \cos x \, dx$) حل

$$dv = \cos x \, dx$$

$$v = \sin x$$

$\frac{1}{2}x^3 \sinh^{-1}\left(\frac{x}{2}\right) -$

$\int -\frac{1}{2} \sinh^{-1}\left(\frac{x}{2}\right) \times 3x^2\, dx$

$\|$

$\vdots$

$\int \frac{x^3}{\sqrt{4+x^2}}\, dx = \int \frac{x^3\, dx}{2\sqrt{1+\frac{x^2}{4}}}$

روش اول

$x^3 = u$

$\int \frac{x^2\, x\, dx}{\sqrt{4+x^2}} = \frac{1}{2}\int \frac{(u-4)\, du}{\sqrt{u}}$

$= \frac{1}{2}\left[\int u^{\frac{1}{2}} du - \int 4 u^{-\frac{1}{2}} du\right] = \dots$

$\int \frac{x^3}{\sqrt{4+x^2}}\, dx$

$4 + x^2 = u$

$du = 2x\, dx$

دو

$$du = (e^{2x} + 2e^{2x}x)\,dx =$$

$$e^{2x}(1+2x)\,dx$$

$$dx = \frac{du}{e^{2x}(1+2x)}$$

u $\quad dv$

$$\int \frac{x e^{2x}}{(1+2x)^2}\,dx$$

$$u = x e^{2x}$$

$$-2\int u e^{u}\,du = -2(u-1)e^{u}$$

$$= -2(\cos x - 1)e^{\cos x} + C$$

$$\cos t = u$$

$$\int e^{\cos t}\sin 2t\,dt =$$

$$-d(\cos t)$$

$$\int 2e^{\cos t}\sin t\cos t\,dt =$$

$$\sin 2t = 2\sin t\cos t$$

$$+C$$

$$\int (\arcsin x)^2\,dx$$

$$\int y\sinh y\,dy$$

$u = \sin x$

$= \int \cos x (1 - \sin^2 x)\, dx$

$= \int (1 - u^2)\, du =$

$u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$

$\int \cos^3 x\, dx =$

$\int \cos x \cos^2 x\, dx$

حل

$\int x \ln(1+x)\, dx$

تمرین

$= \frac{-xe^{2x}}{2(1+2x)} + \int \frac{e^{2x}(1+2x)}{(1+2x)}\frac{dx}{2}$

$= \frac{-xe^{2x}}{2(1+2x)} + \frac{1}{4}e^{2x} + C$

$dv = \frac{1}{(1+2x)^2} dx$ $\quad u = xe^{2x}$ تمرین

$v = \frac{-1}{2(1+2x)}$ $\quad du = e^{2x}(1+2x)\, dx$

$+C$

$= \int (\sin^2 x)^2 (\cos^2 x) \sin x \, dx$

$= \int (1-\cos^2 x)^2 (\cos^2 x) \sin x \, dx$

$\int (1-u^2)^2 u^2 \, du$

$u = \cos x$

$\int \sin^5 x \cos^2 x \, dx$

$\int \sin^m \cos^n \, dx$

$\int \cos^{2n}(x) \, dx$

$\int \cos^{2n+1}(x) \, dx$

$= \frac{1}{2}x + \frac{\sin 2x}{4} + C$

$u = \cos^2 x$

$dv = dx$

$\int = x\cos^2 x + \int x \, 2\cos x \sin x$

$= x\cos^2 x + \int x \sin 2x \, dx$

$+C$

$\int \cos^2 x \, dx = \int \left(\frac{1+\cos 2x}{2}\right) dx$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{\cancel{\cos t}\,dt}{\cancel{\cos t}}$$

$x \in (-1,1)$

$$= t + C$$

$x = \sin t$

$$= \sin^{-1} x + C$$

$dx = \cos t\,dt$

مثال

$$\int \sin^4 x\,dx =$$

$$\int \left(\sin^2 x\right)^2 dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 = \cdots$$

به عهده